1. **The marks of 21 students in a 50 marks mathematics test are given below:**

18	**20**	**25**	**28**	**30**	**35**	**36**	**38**
39	**40**	**41**	**41**	**41**	**42**	**42**	**43**
44	**45**	**45**	**47**	**50**			

a) **Calculate a 10% trimmed mean for the data above.**
b) **Also, calculate the median.**
c) **It was later discovered that the student whose marks were recorded as 35, actually had 45 marks. How will this affect the median value?**

Solution:

a) **10% Trimmed Mean**

To calculate the 10% trimmed mean, we have to eliminate the smallest 10% and the largest 10% of the sample data and then take the average of the remaining observations.

Step 1: Arrange the data in order

Note that as we have to eliminate the smallest and largest 10%, we should first arrange the data in order. In this case, the sample data given is already arranged in ascending order.

Step 2: Find the number of observations to be dropped/ eliminated

n = 21 [Total observations]
10% of n = 10% of 21

$= \frac{10}{100} \times 21 = 2.1$

We have to drop/ eliminate the smallest 2.1 observations and the largest 2.1 observations.

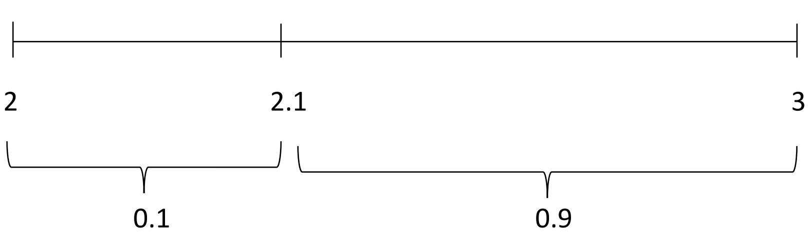
Step 3: Use iteration method (if needed)

The problem is that '2.1' is not an integer. Eliminating smallest and largest 2.1 observations doesn't make sense. So, we will have to use interpolation method here.

We have to try writing 2.1 as a linear combination of 2 integers. As 2.1 lies between 2 and 3, so let us write 2.1 as a linear combination of 2 and 3.

2.1 = a (2) + b (3); where 'a' and 'b' are weights

Trick to find the weights 'a' and 'b':



2.1 is 0.1 away from 2 and 0.9 away from 3. As 2.1 is closer to 2, we should give more weight to 2.

Weight corresponding to the integer '2' = 1 – (Distance of 2.1 from 2)
= 1 – 0.1 = 0.9

i.e. **a = 0.9**

Similarly, weight corresponding to the integer '3' = 1 – (Distance of 2.1 from 3)
= 1 – 0.9 = 0.1

i.e. **b = 0.1**

Let's check our calculation of weights

If our calculation of weights is right, then 2a + 3b should be indeed 2.1.

2a + 3b = 2 (0.9) + 3 (0.1)
= 1.8 + 0.3
= 2.1

⇨ Weight's calculation is right

Step 4: Calculation of Trimmed Mean

Notations:

$\bar{x}_{\left(\frac{2}{21}\right)}$: Trimmed mean when we drop 2 smallest and 2 largest observations

$\left(\frac{2}{21}\right)$: Because we are eliminating 2 smallest and largest observations from 21 observations

Similarly,
$\bar{x}_{\left(\frac{3}{21}\right)}$: Trimmed mean when we drop 3 smallest and 3 largest observations.

Just like 2.1 is the weighted average of 2 and 3, the trimmed mean $\bar{x}_{\left(\frac{2.1}{21}\right)}$ will also be the weighted average of $\bar{x}_{\left(\frac{2}{21}\right)}$ and $\bar{x}_{\left(\frac{3}{21}\right)}$.

2.1 = a (2) + b (3)
where a = 0.9 and b = 0.1

Similarly,

$\bar{x}_{\left(\frac{2.1}{21}\right)} = a\left[\bar{x}_{\left(\frac{2}{21}\right)}\right] + b\left[\bar{x}_{\left(\frac{3}{21}\right)}\right]$; where a = 0.9 and b = 0.1

These are the same weights that we found earlier.

Now to calculate $\bar{x}_{\left(\frac{2.1}{21}\right)}$ we have to first find $\bar{x}_{\left(\frac{2}{21}\right)}$ and $\bar{x}_{\left(\frac{3}{21}\right)}$.

(i) Calculation of $\bar{x}_{\left(\frac{2}{21}\right)}$

To find $\bar{x}_{\left(\frac{2}{21}\right)}$, we have to drop the 2 smallest and largest observations and then take the average of the remaining observations. Dataset after dropping the 2 smallest and largest observations:

25	28	30	35	36	38	39
40	41	41	41	42	42	43
44	45	45				

Note that these are 17 observations.

$$\bar{x}_{\left(\frac{2}{21}\right)} = \frac{25+28+30+35+36+38+39+40+41+41+41+42+42+43+44+45+45}{17}$$

$$\bar{x}_{\left(\frac{2}{21}\right)} = 38.52$$

(ii) Calculation of $\bar{x}_{\left(\frac{3}{21}\right)}$

Dataset after dropping the 3 smallest and largest observations:

28	30	35	36	38	39	40
41	41	41	42	42	43	44
45						

Note that these are 15 observations.

$$\bar{x}_{\left(\frac{3}{21}\right)} = \frac{28+30+35+36+38+39+40+41+41+41+42+42+43+44+45}{15}$$

$$\bar{x}_{\left(\frac{3}{21}\right)} = 39$$

$$\bar{x}_{\left(\frac{2.1}{21}\right)} = 0.9\left[\bar{x}_{\left(\frac{2}{21}\right)}\right] + 0.1\left[\bar{x}_{\left(\frac{3}{21}\right)}\right]$$

$$= 0.9\,(38.52) + 0.1\,(39)$$

$$= 34.668 + 3.9$$

$$= 38.568$$

b) <u>Median</u>

Important: Always order the data/ observations from smallest to largest and then proceed further.

In this case, the sample data is already given in ascending order.

As n is odd (n = 21); median is $\left(\frac{n+1}{2}\right)^{\text{th}}$ term

n = 21, so $\frac{n+1}{2} = \frac{21+1}{2}$ = 11^{th} term.

Note that median is not 11. Median is the 11^{th} term/ observation in our sample data once we have ordered the observations from smallest to largest.

In this case, the 11^{th} observation is 41.

Median = 41

c) **Old/ Original sample data**

18	20	25	28	30	35	36	38
39	40	41	41	41	42	42	43
44	45	45	47	50			

Remove 35 from this data and add 45

New/ Corrected sample data

18	20	25	28	30	36	38	39
40	41	41	41	42	42	43	44
45	45	45	47	50			

Let's find the median using the corrected sample data.
n is still 21, so median = $\left(\frac{n+1}{2}\right)^{\text{th}}$ term = $\left(\frac{21+1}{2}\right)^{\text{th}}$ term = 11^{th} term.
The 11^{th} term in the corrected sample data (already arranged in ascending order) is 41.

So, the median value doesn't change. It is still 41.